Write your name here

Surname **MODEL SOLUTIONS**	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

Advanced
Paper 1: Core Pure Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference **9FM0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Answer ALL questions. Write your answers in the spaces provided.

1. Prove that

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} = \frac{n(an+b)}{12(n+2)(n+3)}$$

where a and b are constants to be found.

(5)

$$\frac{1}{(r+1)(r+3)} = \frac{A}{r+1} + \frac{B}{r+3}$$

$\times (r+1)(r+3)$ $\quad \times (r+1)(r+3)$ $\quad \times (r+1)(r+3)$

$1 = A(r+3) + B(r+1)$

$r = -1: \ 1 = A(2) + B(0)$, $A = \frac{1}{2}$ | $r = -3: \ 1 = A(0) + B(-2)$, $B = -\frac{1}{2}$

1

$$\frac{1}{(r+1)(r+3)} = \frac{1}{2(r+1)} - \frac{1}{2(r+3)}$$

$$\sum_{r=1}^{n} \frac{1}{2(r+1)} - \frac{1}{2(r+3)}$$

$\frac{1}{2(r+1)} = \frac{1}{2(2)}$

$-\frac{1}{2(4)} = -\frac{1}{8}$

$r=1: \ \boxed{\frac{1}{4}} \quad -\cancel{\frac{1}{8}}$

$r=2: \ \boxed{\frac{1}{6}} \quad -\cancel{\frac{1}{10}}$

$r=3: \ \cancel{\frac{1}{8}} \quad -\cancel{\frac{1}{12}}$

$r=4: \ \cancel{\frac{1}{10}} \quad -\frac{1}{14}$

$r=5: \ \cancel{\frac{1}{12}} \quad -\frac{1}{16}$

$\vdots \quad \vdots$

$r=n-1:$

$r=n:$

$r=n+1:$

$r=n+2:$

$r=n-1: \ /$

$r=n: \ /$

$$\boxed{-\frac{1}{2(n+2)} \quad -\frac{1}{2(n+3)}}$$

1

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} = \frac{1}{4} + \frac{1}{6} - \frac{1}{2(n+2)} - \frac{1}{2(n+3)}$$

1

Question 1 continued

$$\frac{1}{4}+\frac{1}{6}-\left(\frac{1}{2(n+3)}+\frac{1}{2(n+2)}\right)$$

$$=\frac{5}{12}-\left(\frac{n+2}{2(n+3)(n+2)}+\frac{(n+3)}{2(n+2)(n+3)}\right)$$

$$=\frac{5}{12}-\left(\frac{2n+5}{2(n+3)(n+2)}\right)$$

$$=\frac{5(n+3)(n+2)}{12(n+3)(n+2)}-\frac{6(2n+5)}{12(n+3)(n+2)}$$

$$=\frac{5(n^2+5n+6)-12n-30}{12(n+3)(n+2)}$$ (1)

$$=\frac{5n^2+25n+\cancel{30}-12n-\cancel{30}}{12(n+3)(n+2)}$$

$$=\frac{5n^2+13n}{12(n+3)(n+2)}$$

$$=\frac{n(5n+13)}{12(n+3)(n+2)}\qquad a=5,\ b=13.$$

$\square$ (1)

(Total for Question 1 is 5 marks)

2. Prove by induction that for all positive integers n,

$$f(n) = 2^{3n+1} + 3(5^{2n+1})$$

is divisible by 17

(6)

Try for $n=1$.

$f(1) = 2^4 + 3(5^3) = 391$

$391 = 17 \times 23$

Thus true for $n=1$. (1)

4 main steps
1) showing true for $n=1$
2) Assume true for $n=k$
3) show also true for $n=k+1$
4) writing a conclusion

Assume true for $n=k$:

$f(k) = 2^{3k+1} + 3(5^{2k+1}) = 17p$, $p \in \mathbb{Z}$. (1)

we need to show $f(k+1)$ is divisible by 17.

$f(k+1) = 2^{3(k+1)+1} + 3(5^{2(k+1)+1})$ (1)

$= 2^{3k+3+1} + 3(5^{2k+2+1})$ $\qquad a^{b+c} = a^b \times a^c$

$= 2^{3k+1} \times 2^3 + 3(5^{2k+1} \times 5^2)$

$= 2^{3k+1} \times 8 + 3(5^{2k+1} \times 25)$ $\quad 8+17$

$= 2^{3k+1} \times 8 + 3(5^{2k+1} \times (8+17))$

$= 2^{3k+1} \times 8 + 3(5^{2k+1}) \times 8 + 3(5^{2k+1})(17)$

$= 8(\underbrace{2^{3k+1} + 3(5^{2k+1})}) + 17(3(5^{2k+1}))$

$= \underbrace{8(f(k))} + \underbrace{17(3(5^{2k+1}))}_{\text{divisible by 17}}$ (1)

$= 8(17p) + 17(3(5^{2k+1})) = f(k+1)$

$\Rightarrow f(k+1) = 17(8p + 3(5^{2k+1}))$ (1)

$\therefore f(k+1)$ is divisible by 17

- Proven statement is true for $n=1$
- Shown that when statement is assumed true for $n=k$, also true for $n=k+1$.
- Thus, BY INDUCTION, true for all $n \in \mathbb{Z}^+$.

(1)

3.
$$f(z) = z^4 + az^3 + \boxed{6z^2} + bz + 65$$

where a and b are real constants.

Given that $z = 3 + 2\mathbf{i}$ is a root of the equation $f(z) = 0$, show the roots of $f(z) = 0$ on a single Argand diagram.

(9)

$z = 3+2i$, $z = 3-2i$ (1)

$(z-(3+2i))(z-(3-2i))$ (1)

$(3+2i)(3-2i)$
$= 9-(2i)^2$
$= 9-(-4) = 13$

$= z^2 - z(3+2i) - z(3-2i) + 13$

$= z^2 - z(6) + 13$ (1)

$f(z) = (z^2-6z+13)(\alpha z^2+\beta z+\gamma)$

$13\gamma = 65$

$\gamma = 5$

$z^4 = \alpha z^2 \times z^2$

$\alpha = 1$

$f(z) = (z^2-6z+13)(z^2+\beta z+5)$ (1)

6 is the coefficient of z^2 for $f(z)$

$5z^2 - 6\beta z^2 + 13z^2 = 6z^2$

$5 - 6\beta + 13 = 6$

$12 = 6\beta$

$\beta = 2$

(1)

$f(z) = (z^2-6z+13)(z^2+2z+5) = 0$

$z = 3 \pm 2i$

completing the square:

$(z+1)^2 - 1 + 5 = 0$

$(z+1)^2 = -4$ (1)

$z+1 = \pm 2i$

$z = -1 \pm 2i$ (1)

$z = 3+2i,\ 3-2i,\ -1+2i,\ -1-2i$

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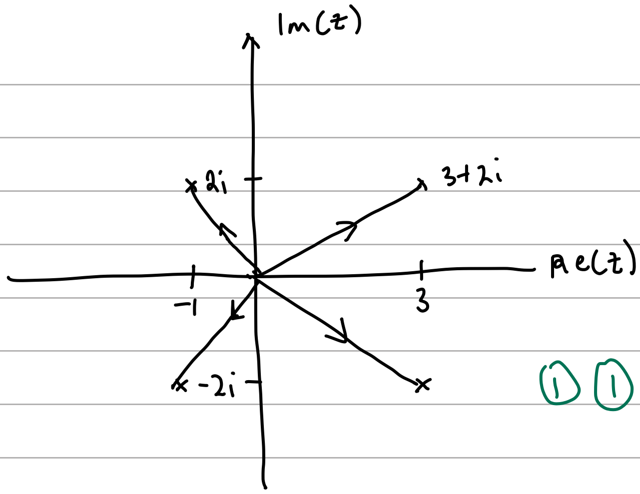
Question 3 continued

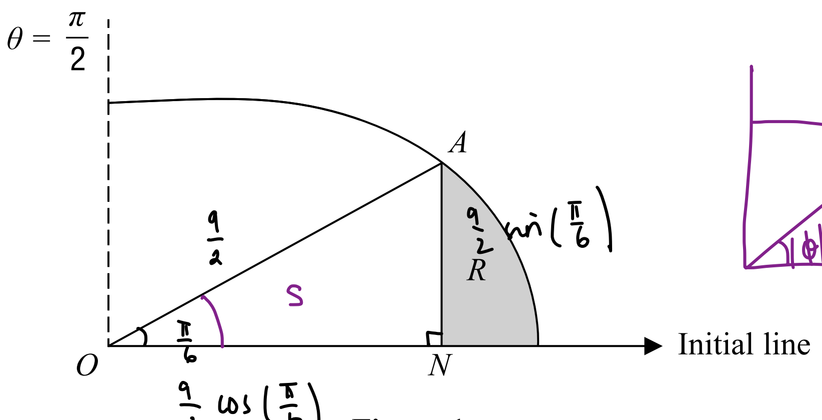


$3 \pm 2i$

$-1 \pm 2i$

(Total for Question 3 is 9 marks)

4.



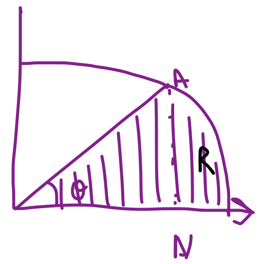


Figure 1

The curve C shown in Figure 1 has polar equation

O A N $\cdot \ \frac{1}{2}\left(\frac{9}{2}\cos\left(\frac{\pi}{6}\right)\times\frac{9}{2}\sin\left(\frac{\pi}{6}\right)\right)$

$$r = 4 + \cos 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{9}{2}$

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(9)

• polar integration

$\frac{9}{2} = 4 + \cos 2\theta$

$\frac{1}{2} = \cos 2\theta$

$2\theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6}$ (1) (1)

area of polar curve:

$\frac{1}{2}\int_{\alpha}^{\beta} (r)^2 \, d\theta$

$S + R = \frac{1}{2}\int_0^{\frac{\pi}{6}} (4 + \cos 2\theta)^2 \, d\theta$ (1)

$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (16 + 8\cos 2\theta + \cos^2 2\theta) \, d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \left(16 + 8\cos 2\theta + \frac{\cos(4\theta)+1}{2}\right) d\theta$ (1)

$= \frac{1}{2}\left[16\theta + 8\left(\frac{\sin 2\theta}{2}\right) + \frac{\sin(4\theta)}{8} + \frac{\theta}{2}\right]_0^{\frac{\pi}{6}}$ (1)

$4 \quad + \cos 2\theta$

$4 \quad 16 \quad + 4\cos 2\theta$

$+\cos 2\theta + 4\cos 2\theta \quad + \cos^2 2\theta$

$\cos 2A = 2\cos^2 A - 1$

$\frac{\cos 2A + 1}{2} = \cos^2 A$

$\frac{\cos(4\theta)+1}{2} = \cos^2(2\theta)$

$$\sin\left(\frac{\pi}{6}\times 4\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

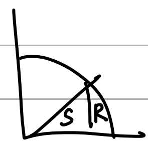
$$= \frac{1}{2}\left[\frac{16\pi}{6} + 8\left(\frac{\sqrt{3}}{4}\right) + \frac{1}{8}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{12}\right]$$

$$= \frac{1}{2}\left[\frac{11\pi}{4} + \frac{33\sqrt{3}}{16}\right] \quad 1$$

$$= \frac{11\pi}{8} + \frac{33\sqrt{3}}{32}$$

area of triangle OAN:

$$S = \frac{1}{2}\left(\left(\frac{9}{2}\right)^2 \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right)\right) \quad 1$$



$$= \frac{81\sqrt{3}}{32}$$

$$R = \frac{11\pi}{8} + \frac{33\sqrt{3}}{32} - \frac{81\sqrt{3}}{32} \quad 1$$

$$\frac{11\pi}{8} - \frac{3\sqrt{3}}{2} \qquad p = \frac{11}{8}, \quad q = -\frac{3}{2} \quad 1$$

(Total for Question 4 is 9 marks)

5. A pond initially contains 1000 litres of unpolluted water.

The pond is leaking at a constant rate of 20 litres per day.

It is suspected that contaminated water flows into the pond at a constant rate of 25 litres per day and that the contaminated water contains 2 grams of pollutant in every litre of water.

It is assumed that the pollutant instantly dissolves throughout the pond upon entry.

Given that there are x grams of the pollutant in the pond after t days,

(a) show that the situation can be modelled by the differential equation,

$$\frac{dx}{dt} = 50 - \frac{4x}{200 + t}$$

rate of change of pollutant in pond with respect to t

(4)

(b) Hence find the number of grams of pollutant in the pond after 8 days.

(5)

(c) Explain how the model could be refined.

2g of pollutant flows in with every litre of water.

(1)

a) Initial volume of water : 1000 L

Rate of flow in: 25 L/D
Rate of flow out: 20 L/D
Total flow into pond: 5 L/D

Total volume of water after t days:
1000+5t Litres after t days 1

so $\frac{dx}{dt}$ = (rate of pollutant in) - (rate of pollutant out)

rate of pollutant in: $25 \text{ L/D} \times 2 \text{ g/L} = 50 \text{ g/D}$ 1

rate of pollutant out: $20 \text{ L/D} \times \frac{x}{1000+5t} \text{ g/L}$ 1

$\frac{x}{1000+5t}$ g/L

$\Rightarrow \frac{dx}{dt} = 50 - \frac{4x}{200+t}$ 1

$\square$

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Question 5 continued

b) $\frac{dx}{dt} = 50 - \frac{4x}{200+t}$

$\frac{dx}{dt} + P(t)x = Q(t)$

$I(t) = e^{\int P(t)dt}$

$\frac{dx}{dt} + \frac{4}{200+t}x = 50$

$I(t) = e^{\int \frac{4}{200+t} dt}$

$I(t) = e^{4\ln|200+t|}$
$= e^{\ln(200+t)^4}$
$= (200+t)^4$

A pond initially contains 1000 litres of unpolluted water.

$t = 0$ $\quad x = 0$

$\frac{d}{dt}\left((200+t)^4 x\right) = 50(200+t)^4$

$(200+t)^4 x = \int 50(200+t)^4 \, dt$ (1)

$(200+t)^4 x = 10(200+t)^5 + c$ (1)

$x = 10(200+t) + \frac{c}{(200+t)^4}$

$0 = 10(200+0) + \frac{c}{(200+0)^4}$

$c = [-10(200)][200^4]$
$c = -3.2 \times 10^{12}$ (1)

$x = 10(200+t) - \frac{3.2\times10^{12}}{(200+t)^4}$ (1)

$x = 10(200+8) - \frac{3.2\times10^{12}}{(200+8)^4} \approx 370\text{g}$ (1)

c) - The rate of leaking could be made to vary with the volume of water in the pond. (1)

ALTERATE ANSWER

- The model should take into account that the pollutant does not dissolve throughout the pond upon entry. (1)

6.

$$f(x) = \frac{x+2}{x^2+9}$$

(a) Show that

$$\int f(x)dx = A\ln(x^2+9) + B\arctan\left(\frac{x}{3}\right) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence show that the mean value of f(x) over the interval [0, 3] is

$$\frac{1}{6}\ln 2 + \frac{1}{18}\pi \qquad 0 \leq x \leq 3$$

(3)

(c) Use the answer to part (b) to find the mean value, over the interval [0, 3], of

$$f(x) + \ln k$$

where k is a positive constant, giving your answer in the form $p + \frac{1}{6}\ln q$, where p and q are constants and q is in terms of k.

(2)

a) $\int f(x)\,dx = \int \frac{x+2}{x^2+9}\,dx$

$= \int \frac{x}{x^2+9} + \frac{2}{x^2+9}\,dx$ 1 $\qquad \frac{1}{a^2+x^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$ $\quad a = 3$

$= \frac{1}{2}\ln|x^2+9| + \frac{2}{3}\arctan\left(\frac{x}{3}\right) + c$ 1 1

$A = \frac{1}{2},\ B = \frac{2}{3}.$ 1

interval: $0 \leq x \leq 3$

b) $\frac{1}{b-a}\int_a^b f(x)\,dx$

$\frac{1}{3-0}\int_0^3 f(x)\,dx$

$= \frac{1}{3}\left[\frac{1}{2}\ln|x^2+9| + \frac{2}{3}\arctan\left(\frac{x}{3}\right)\right]_0^3$

$= \frac{1}{3}\left[\frac{1}{2}\ln(18) + \frac{2}{3}\arctan(1) - \frac{1}{2}\ln(9) + 0\right]$

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Question 6

$= \frac{1}{3}\left[\frac{1}{2}\ln(18) + \frac{2}{3}\left(\frac{\pi}{4}\right) - \frac{1}{2}\ln 9\right]$ 1

$18 = 9 \times 2$

$\ln(18) = \ln(9) + \ln(2)$

$= \frac{1}{6}\ln(18) + \frac{\pi}{18} - \frac{1}{6}\ln(9)$

$= \cancel{\frac{1}{6}\ln 9} + \frac{1}{6}\ln 2 + \frac{\pi}{18} - \cancel{\frac{1}{6}\ln(9)}$ 1

$= \frac{1}{6}\ln 2 + \frac{\pi}{18}$ 1

c) $\frac{1}{6-9}\int_0^3 f(x) + \ln k \, dx$

$\frac{1}{3}\int_0^3 f(x)\,dx + \frac{1}{3}\int_0^3 \ln k \, dx$

$= \frac{1}{6}\ln 2 + \frac{\pi}{18} + \frac{1}{3}\left[x\ln k\right]_0^3$

$= \frac{1}{6}\ln 2 + \frac{\pi}{18} + \frac{1}{3}\left[3\ln k\right]$

$= \frac{1}{6}\ln 2 + \frac{\pi}{18} + \ln k$ 1

$= \frac{1}{6}\left[\ln 2 + \ln k^6\right] + \frac{\pi}{18}$

$= \frac{1}{6}\ln(2k^6) + \frac{\pi}{18}$ 1

(Total for Question 6 is 9 marks)

7.

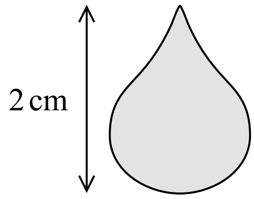


Figure 2

Figure 2 shows the image of a gold pendant which has height 2 cm. The pendant is modelled by a solid of revolution of a curve C about the y-axis. The curve C has parametric equations

$$x = \cos\theta + \frac{1}{2}\sin 2\theta, \qquad y = -(1 + \sin\theta) \qquad 0 \leqslant \theta \leqslant 2\pi$$

(a) Show that a Cartesian equation of the curve C is

$$x^2 = -(y^4 + 2y^3)$$

(4)

(b) Hence, using the model, find, in cm^3, the volume of the pendant.

(4)

a) $x = \cos\theta + \frac{1}{2}\sin 2\theta$ $\qquad$ $\sin 2\theta = 2\sin\theta\cos\theta$

$x = \cos\theta + \frac{1}{2}(2\sin\theta\cos\theta)$

$x = \cos\theta + \sin\theta\cos\theta$

$x = \cos\theta(1 + \sin\theta)$ $\qquad$ $-y = 1 + \sin\theta$ $\qquad$ $\sin\theta = -1 - y$

$x = \cos\theta(-y)$ 1 $\qquad$ $= -(1+y)$

1

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

$\cos\theta = \pm\sqrt{1 - (-(1+y))^2}$

$\cos\theta = \pm\sqrt{1 - (1 + 2y + y^2)}$

$\cos\theta = \pm\sqrt{-(2y + y^2)}$

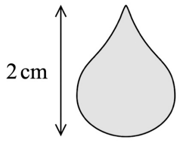
Plug into our original expression for x

$\Rightarrow x = \pm\sqrt{-(2y + y^2)}\,(-y)$

$\Rightarrow x^2 = -(2y + y^2)(y^2)$ 1

$\Rightarrow x^2 = -(2y^3 + y^4)$

$\Rightarrow x^2 = -(y^4 + 2y^3)$ 1 $\square$



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$$x^2 = -(y^4 + 2y^3)$$

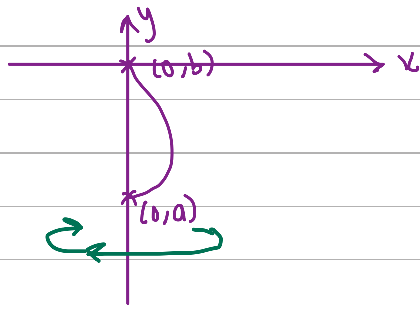
(4)

(b) Hence, using the model, find, in cm³, the volume of the pendant.

(4)

Volume of revolutions formula

$= \pi \int_a^b x^2 \, dy$ (around y axis)



b) $0 = -(y^4 + 2y^3)$

$0 = y^3(y+2)$

$\Rightarrow y = 0, \; y = -2$

volume of pendant:

$= \pi \int_{-2}^{0} x^2 \, dy$

$= \pi \int_{-2}^{0} -(y^4 + 2y^3) \, dy$ 1

$= -\pi \int_{-2}^{0} y^4 + 2y^3 \, dy$

$= -\pi \left[\frac{1}{5}y^5 + \frac{1}{2}y^4 \right]_{-2}^{0}$ 1

$= -\pi \left[\frac{1}{5}(0) + \frac{1}{2}(0) - \frac{1}{5}(-2)^5 - \frac{1}{2}(-2)^4 \right]$ 1

$= -\pi \left[-1.6 \right]$

$= \underline{1.6\pi \text{ cm}^3} \approx \underline{5.03 \text{cm}^3}$ 1

8. The line l_1 has equation $\dfrac{x-2}{4} = \dfrac{y-4}{-2} = \dfrac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π.

Find a vector equation of the line l_2

(7)

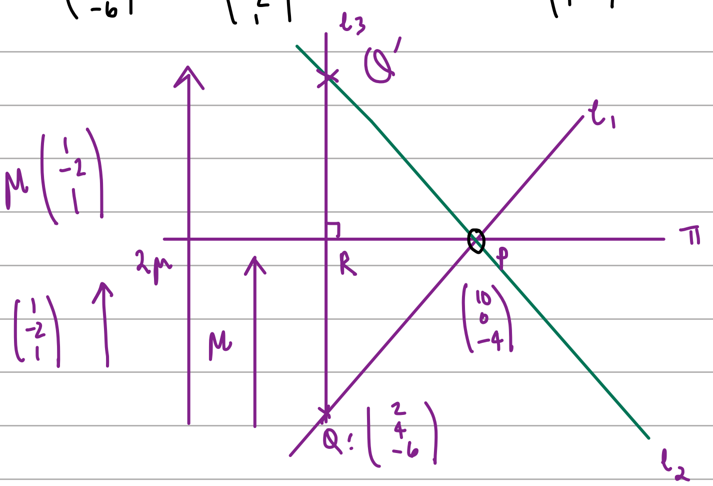
cartesian form of a line: $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$ ∴ of a plane: $ax+by+cz=k$

column form of a line: $\underline{r} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ ∴ of plane: $\underline{r} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = k$

normal vector

$l_1: \quad \underline{r} = \begin{pmatrix} 2 \\ 4 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \qquad \Pi: \underline{r} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 6$

$l_3: \underline{r} = \begin{pmatrix} 2 \\ 4 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$



• minimum of two points in order to find vector equation for l_2.

$\underline{r} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \boxed{\begin{pmatrix} a \\ b \\ c \end{pmatrix}}$

we need two points to find this.

$l_1: \underline{r} = \begin{pmatrix} 2+4\lambda \\ 4-2\lambda \\ -6+\lambda \end{pmatrix} \qquad \Pi: \underline{r} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 6$

$\begin{pmatrix} 2+4\lambda \\ 4-2\lambda \\ -6+\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 6$

$(2+4\lambda)(1) - 2(4-2\lambda) + 1(-6+\lambda) = 6$

$2 + 4\lambda - 8 + 4\lambda - 6 + \lambda = 6$

$9\lambda = 18$

$\lambda = 2$ (1)

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Question 8 continued

$$\Rightarrow \quad p: \begin{pmatrix} 2+4(2) \\ 4-2(2) \\ -6+(2) \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} \quad 1$$

$$l_3: \underline{r} = \begin{pmatrix} 2 \\ 4 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$l_3: \underline{r} = \begin{pmatrix} 2+\mu \\ 4-2\mu \\ -6+\mu \end{pmatrix} \qquad \Pi: \underline{r} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 6$$

$$\begin{pmatrix} 2+\mu \\ 4-2\mu \\ -6+\mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 6$$

$$2+\mu - 2(4-2\mu) - 6 + \mu = 6$$

$$2+\mu - 8 + 4\mu - 6 + \mu = 6$$

$$6\mu = 18$$

$$\mu = 3 \quad 1$$

$$3 \times 2 = 6 = 2\mu$$

$$Q': \begin{pmatrix} 2+(6) \\ 4-2(6) \\ -6+(6) \end{pmatrix} \quad 1$$

$$Q': \begin{pmatrix} 8 \\ -8 \\ 0 \end{pmatrix} \quad 1$$

Question 8 continued

$P: \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix}$, $Q': \begin{pmatrix} 8 \\ -8 \\ 0 \end{pmatrix}$

$\overrightarrow{PQ'}: \begin{pmatrix} 8 \\ -8 \\ 0 \end{pmatrix} - \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -8 \\ 4 \end{pmatrix}$ (1)

$l_2: \underline{r} = \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ -8 \\ 4 \end{pmatrix}$ (1) //

9. A company plans to build a new fairground ride. The ride will consist of a capsule that will hold the passengers and the capsule will be attached to a tall tower. The capsule is to be released from rest from a point half way up the tower and then made to oscillate in a vertical line.

The vertical displacement, x metres, of the top of the capsule below its initial position at time t seconds is modelled by the differential equation,

$$m\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + x = 200\cos t, \qquad t \geqslant 0$$

where m is the mass of the capsule including its passengers, in thousands of kilograms.

The maximum permissible weight for the capsule, including its passengers, is 30 000 N.

Taking the value of g to be $10\,\text{m s}^{-2}$ and assuming the capsule is at its maximum permissible weight,

$W = Mg$

(a) (i) explain why the value of m is 3

(ii) show that a particular solution to the differential equation is

$$x = 40\sin t - 20\cos t$$

(iii) hence find the general solution of the differential equation.

(8)

(b) Using the model, find, to the nearest metre, the vertical distance of the top of the capsule from its initial position, 9 seconds after it is released.

(4)

ai) $W = Mg$

$30000 = M \times 10$

$\Rightarrow M = 3000$

$\Rightarrow m = M / 1000 = 3$

thus $m = 3$ // (1)

ii) $x = 40\sin t - 20\cos t$

$\frac{dx}{dt} = 40\cos t + 20\sin t$ (1)

$\frac{d^2x}{dt^2} = -40\sin t + 20\cos t$

$3\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + x$

$= 3(-40\sin t + 20\cos t) + 4(40\cos t + 20\sin t) + (40\sin t - 20\cos t)$ (1)

$= -120\sin t + 60\cos t + 160\cos t + 80\sin t + 40\sin t - 20\cos t$

Question 9 continued 1

$-200\cos t$ so particular integral : $40\sin t - 20\cos t$

iii) Our DE: $3\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + x = 200\cos t$

AE: $3\lambda^2 + 4\lambda + 1 = 0$

$(\lambda+1)(3\lambda+1) = 0$

$\lambda = -1, \ \lambda = -\frac{1}{3}$ 1

discriminant of AE: $4^2 - 3(1)(4)$

$= 4$

$4 > 0$

general equation will be in the form

$x = Ae^{\alpha t} + Be^{\beta t}$ + (our particular integral)

$x_c = Ae^{-t} + Be^{-\frac{1}{3}t}$ 1 (our complementary function)

but $x = CF + PI$ 1

$\Rightarrow x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40\sin t - 20\cos t$ // 1

A company plans to build a new fairground ride. The ride will consist of a capsule that will hold the passengers and the capsule will be attached to a tall tower. The capsule is to be released from rest from a point half way up the tower and then made to oscillate in a vertical line.

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where m is the mass of the capsule including its passengers, in thousands of kilograms.

The maximum permissible weight for the capsule, including its passengers, is 30000 N.

Taking the value of g to be 10 ms^{-2} and assuming the capsule is at its maximum permissible weight, $W = Mg$

(a) (i) explain why the value of m is 3

(ii) show that a particular solution to the differential equation is

$$x = 40\sin t - 20\cos t$$

(iii) hence find the general solution of the differential equation.

(8)

(b) Using the model, find, to the nearest metre, the vertical distance of the top of the capsule from its initial position, 9 seconds after it is released.

(4)

b) • we need initial conditions

$t = 0$

$x = 0$

$\frac{dx}{dt} = 0$

Question 9 continued

$x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40\sin t - 20\cos t$

$t = 0,\ x = 0,\ \frac{dx}{dt} = 0$

$0 = Ae^{0} + Be^{0} - 20$

$① \Rightarrow\ 20 = A + B$ ①

$\frac{dx}{dt} = -Ae^{-t} - \frac{1}{3}Be^{-\frac{1}{3}t} + 40\cos t + 20\sin t$

$0 = -A - \frac{1}{3}B + 40$

$② \Rightarrow\ A + \frac{1}{3}B = 40$ ①

$① - ②$

$A + B = 20$

$A + \frac{1}{3}B = 40$

$\frac{2}{3}B = -20$

$B = -30$

subbing back into ①:

$20 = A - 30$

$A = 50$

$x = 50e^{-t} - 30e^{-\frac{1}{3}t} + 40\sin t - 20\cos t$ ①

$t = 9,\ x = 33\text{m}$ ①